

Quantitative Finance

Discount Interest – Chap. 2



Simple Discount or Bank Discount

In a **simple discount** interest is based on the future value, with interest paid at the beginning of the term.

The money that is borrowed is called the **amount** (future value).

The percentage used to compute the interest charges is called the **discount rate**.

The interest charged for the use of borrowed money is called the **discount**.

The money received from the loan is called **proceeds or principal** (present value).

Simple Discount or Bank Discount

$$\text{Discount} = \text{discount rate} \times \text{time} \times \text{Amount}$$

Principal
(P)

Amount
(FV)

Proceeds (or Principal) = Amount – Discount.

Principal is the money that the borrower receives at the moment zero.

Formula: The Simple Discount Formula

$$D = FV \times d \times t$$

D represents the amount of **SIMPLE DISCOUNT** for a loan (equivalent to interest)

FV represents the **MATURITY VALUE**

d represents the **INTEREST DISCOUNT RATE**

t represents the time

Formula: The Simple Discount Formula

$$D = FV \times d \times t \Rightarrow$$

$$P = FV - D$$

$$\Leftrightarrow P = FV - FVdt$$

$$\Leftrightarrow \mathbf{P = FV (1-dt)}$$
 moves money backward

$$\Leftrightarrow \mathbf{FV = P/(1-dt)}$$
 moves money forward

Examples of Simple discount instruments:

- **Promissory note:** is a document on which one party writes his or her promise to pay another party the principal and the interest for a loan due at some date in the future.
- **Treasury Bills:** are short-term loans to the U.S. federal government, carrying terms ranging from a few days to 6 months, though the most common terms are 4, 13, or 26 weeks.
- **Commercial paper:** is a type of promissory note used by corporations, and is short-term (the term is lower than 12 months). Only corporations with high-quality ratings will easily find buyers for this type of instrument-

All those instruments are all sold at a discount.

Simple Discount (other examples)

Example 1

Jennifer knows she will be getting a paycheck for €500 at the end of the week, but she needs money now. She takes out a loan against this paycheck, borrowing as much as she can pay off with her check when it arrives.

Simple Discount (other examples)

Example 2

When you file your income taxes, you find that you are owed a refund of €737,15. Your tax consultant offers the option of getting your money right away, instead of waiting for the IRS to process your return and send your refund check. In exchange for agreeing to sign over your refund check when it arrives, the IRS consultant agrees to lend you €707,15 today.

Simple Discount (other examples)

Example 3

The XZ Hospital is scheduled to receive a state aid check for €72.500 on April 1. The Hospital needs the funds by mid-February to meet expenses, but unfortunately the state is unwilling to make the payment early. In order to avoid a cash crunch, the Hospital borrows €71.342 from the Bank, to be repaid on April 1 when the state aid arrives. The amount borrowed is based on the €72.500 that the Hospital will have available for repayment when the aid check arrives.

Simple Discount

- If we look at the XZ Hospital example, we would call:
 - €71.342 is the principal
 - $€72.500 - €71.342 = €1.158$ the discount
 - €72.500 (principal + interest) the amount (future or maturity value)
 - April 1 the maturity date

Simple Discount

- One common example of a discount note is U.S. Savings Bonds (long term). A savings bond is actually a promissory note issued by the U.S. government so, when you buy a savings bond, you are actually loaning money to the USA State.
- Although there are several types of savings bonds, the most familiar type is sold for half of its face value.
- Therefore, a “€50 U.S. Savings Bond” actually costs only €25. The €50 face value reflects the amount the government guarantees that it will pay for the bond on maturity, which is usually many years away (usually 20 years).

Simple Discount

- Treasury bills (known as T bills) are another common example of a discount note. They are short-term loans to the U.S. federal government, carrying terms ranging from a few days to 6 months, though the most common terms are 4, 13, or 26 weeks.
- You might buy a €1000 face value T bill for €985, a €15 discount from the maturity value.

Simple Discount

Example

A €10.000 face value discount note has a term of 4 months. The simple discount rate is 6%. Find the amount of the discount.

- Solution

$$D = FV d t$$

$$D = €10.000 \times 6\% \times 4/12$$

$$D = €10.000 \times 0,06 \times 4/12$$

$$D = €200$$

Simple Discount

Example

A €5000 face value note has a term of 219 days (exact interest). The simple discount rate is 9,375%. Find the principal of the note.

- Solution

$$D = FV d t$$

$$D = €5000 \times 9,375\% \times 219/365$$

$$D = €281,25$$

$$\text{Principal} = FV - \text{Discount}$$

$$\text{Principal} = €5.000 - €281,25 = €4718,75$$

Comparing Simple and Discount Interest

1. Because simple interest is based on present value and discount is based on future value, discount interest costs the borrower more (when the interest rate is positive).
2. If the interest charges were the same, then the interest rates would have to be different, and the simple interest rate would be the higher.
3. This simple interest rate is called the coupon equivalent. Here the word equivalent implies that they produce the same interest amount.

Comparing Simple and Discount Interest

- Discount Interest Loan
- Simple Interest Loan

- $P = FV (1-dt)$

- $P = FV/(1+it)$

- $FV(1-dt) = FV/(1+it)$

- $1-dt = 1/(1+it)$

- $1+it = 1/(1-dt)$

- $it = 1/(1-dt) - 1$

- $it = (1-(1-dt))/(1-dt)$

- $it = dt/(1-dt)$

(coupon equivalent) $i = d/(1-dt)$ or solving for $d = i/(1+i)$ $\Leftrightarrow i > d$

Simple Discount vs. Simple Interest

- Suppose that Mr. Jack borrow €38.000 for one year from Bank KK. The maturity value of the note was €40.000. Find the simple interest rate and the simple discount rate.

- Simple interest case:**

Principal = €38.000

Interest = €2.000

Maturity Value = €38.000 + €2.000 = €40.000

$I = Pit \Leftrightarrow €2.000 = €38.000 \times i \times 1$

$i = 0,0526 = 5,26\%$

- Simple discount?**

Maturity Value = €40.000

Discount = €2.000

Principal = €38.000

$D = FVdt \Leftrightarrow €2.000 = €40.000 \times d \times 1$

$d = 0,05 = 5\%$

Simple Discount vs. Simple Interest

Example

Mr. Jack invested €49.200 in bonds whose maturity values totaled €50.000. The remaining term of the bonds was 146 days. The simple discount rate was 4%, but what would the equivalent simple interest rate be? (use the exact interest):

- Solution

$$I = P \times i \times t$$

$$I = D = €800 = €49.200 \times i \times 146/365$$

$$R = 4,07\%$$